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On the Rate of Aeolian Sand Transport

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Introduction

During the last 15 years a number of numerical models of aeolian sand transport have been published (Anderson & Haff, 1988; Werner, 1990; McEwan & Willetts, 1991; Spies & McEwan, 2000) that incorporate the present understanding of the physics of sand transport by wind, see Anderson, Sørensen, & Willetts (1991). An analytic model of aeolian saltation based on essentially the same physics (Sørensen, 1991) resulted in a formula for the transport rate that seems to compare well with transport rate data (Rasmussen & Mikkelsen, 1991); see also the discussion in McEwan & Willetts (1994).

Here the results in Sørensen (1991) are improved significantly in two directions. By introducing into the theory a hypothesis by Owen (1964), a more satisfactory and simpler model is obtained. Moreover, the derivation of the formula for the transport rate is far simpler and more lucid. In particular, the constants in the formula have a physical interpretation. The new formula is calibrated to wind-tunnel data by Iversen and Rasmussen (1999).

The wind profile during saltation

The following theory of how the cloud of saltating grains modifies the wind profile is based on the concepts of eddy viscosity and grain borne shear stress. The latter was introduced by Owen (1964). The grain borne shear stress at height y, $T(y)$, is the part of the shear stress that at height y is carried by the saltating grains. It can be calculated as (Sørensen, 1985)

$$T(y) = \Phi v(y),$$

where $v(y)$ is the average increase of the horizontal velocity component of a saltating grain while it is above the level y, and where Φ is the mass of grains that on average leaves one unit of surface area per time unit. The air borne shear stress at height y is equal to $\rho U_*^2 - T(y)$, where U_* denotes the friction speed and ρ is the density of air. This is the difference between the shear stress in the grain free wind above the saltation cloud and the grain borne shear stress at heigh y. Owen (1964) argued that at all friction speeds, the air borne shear stress at the surface is equal to its value at the impact threshold U_{*c}, i.e. it is equal to ρU_{*c}^2. If this is true, we find that

$$\Phi = \rho U_*^2 \left(1 - V^{-2}\right) / v(0), \tag{1}$$

where $V = U_*/U_{*c}$.

Next we calculate how the wind profile is modified by the saltating grains based on eddy viscosity theory. We assume that the eddy viscosity is given by

$$\nu(y) = \kappa y \sqrt{U_*^2 - T(y)/\rho} = \kappa y U_* \sqrt{1 - \left(1 - V^{-2}\right) v(y)/v(0)}, \tag{2}$$

where κ is von Kármán's constant. This is analogous to the usual derivation of the logarithmic wind profile and was first proposed by Anderson (1986). The quantity $\sqrt{U_*^2 - T(y)/\rho}$ is an equivalent friction speed corresponding to the air borne shear stress at height y. The expression (2) implies that the wind profile $U(y)$ satisfies

$$\frac{dU}{dy} = \frac{U_*}{\kappa y}\sqrt{1 - (1 - V^{-2})\, v(y)/v(0)}. \tag{3}$$

This equation does not yield an explicit expression for the wind profile, but if we approximate $\sqrt{1-x}$ by $1-x$ for x between zero and one, we find that

$$U(y) = \kappa^{-1} U_* \left[\ln(y/y_0) - \left(1 - V^{-2}\right) b(y)\right], \tag{4}$$

where y_0 is the height at which the wind speed is zero, and where the dimensionless function b is given by

$$b(y) = \int_{y_0}^{y} z^{-1} \frac{v(z)}{v(0)} dz. \tag{5}$$

An alterative derivation of the expression (4) for the wind profile in the saltation layer was made in Sørensen (1985) by assuming that the eddy viscosity is $\nu(z) = \kappa z U_*$.

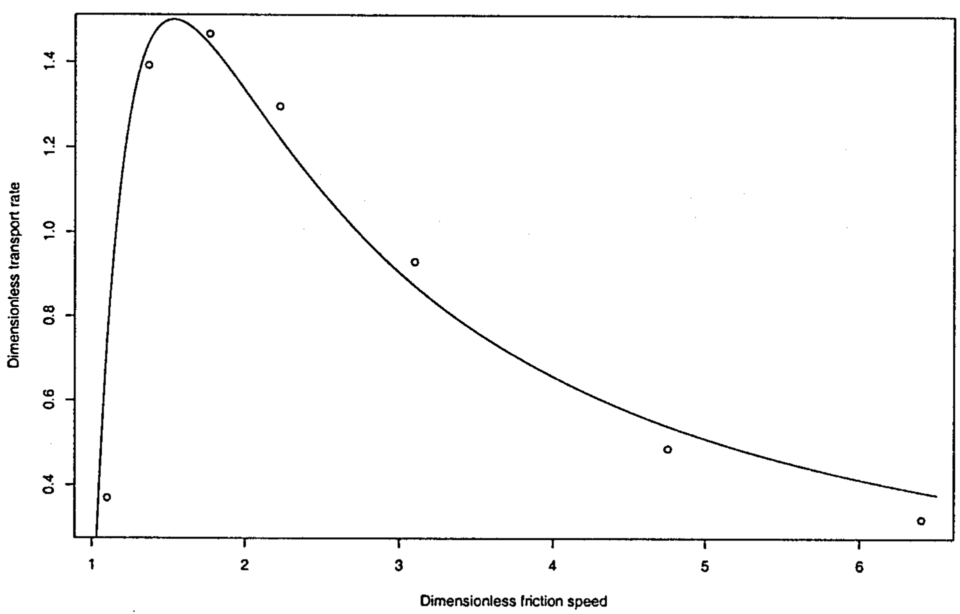

Figure 1: The dimensionless transport rate $Qg/(\rho U_*^3)$ plotted agains the dimensionless friction speed $V = U_*/U_{*c}$ for the experiment with homogeneous sand of size 170 μm (indicated by $\circ$). The curve is the transport rate formula (7) with $\alpha = 0$, $\beta = 2.98$, and $\gamma = 2.06$.

The transport rate

The transport rate Q is given by

$$Q = \Phi \bar{\ell}, \tag{6}$$

where $\bar{\ell}$ denotes the mean jump length of a saltating grain, see Sørensen (1985). By combining the analytic model for the saltation trajectories in Sørensen (1991) (based on Owen's (1964) linear drag approximation) with the wind profile (4), an expression for $\bar{\ell}$ can be obtained, and using (1), we find that the dimensionless transport rate is given by

$$\frac{Qg}{\rho U_*^3} = \left(1 - V^{-2}\right)\left[\alpha + \beta V^{-2} + \gamma V^{-1}\right] \tag{7}$$

where g denotes the acceleration of gravity. The quantities α, β and γ have the following interpretation: $\alpha v(0)/g$ is the average jump length of a grain that starts vertically in the wind profile $\kappa^{-1}[\ln(y/y_0) - b(y)]$, $\beta v(0)/g$ is the average jump length of a grain that starts vertically in the wind profile $\kappa^{-1}b(y)$, while $\gamma v(0)U_{*c}/g$ is the average jump length of a grain if it were not accellerated by the wind. A simple formula for the transport rate is thus obtained if the quantities α, β and γ do not depend on the friction velocity. They will obviously depend on the type of sand being transported, e.g. on the grain size. Note that $v(0)/g$ is of the order of the typical duration of a saltation jump.

Calibration to wind-tunnel data

The transport rate formula (7) was fitted to data obtained in wind tunnel experiments by Iversen and Rasmussen (1999). Figure 1 shows a plot of the data from an experiment using very homogeneous sand with a typical size of 170 μm and the fitted curve. The formula fits the data well. A slightly better fit can be obtained if α is given a negative value. Data from an experiment using natural sand with a typical grain size of 230 μm are plotted in Figure 2. Here a negative value of α has been allowed since the fit for the largest friction speed was otherwise less satisfactory.

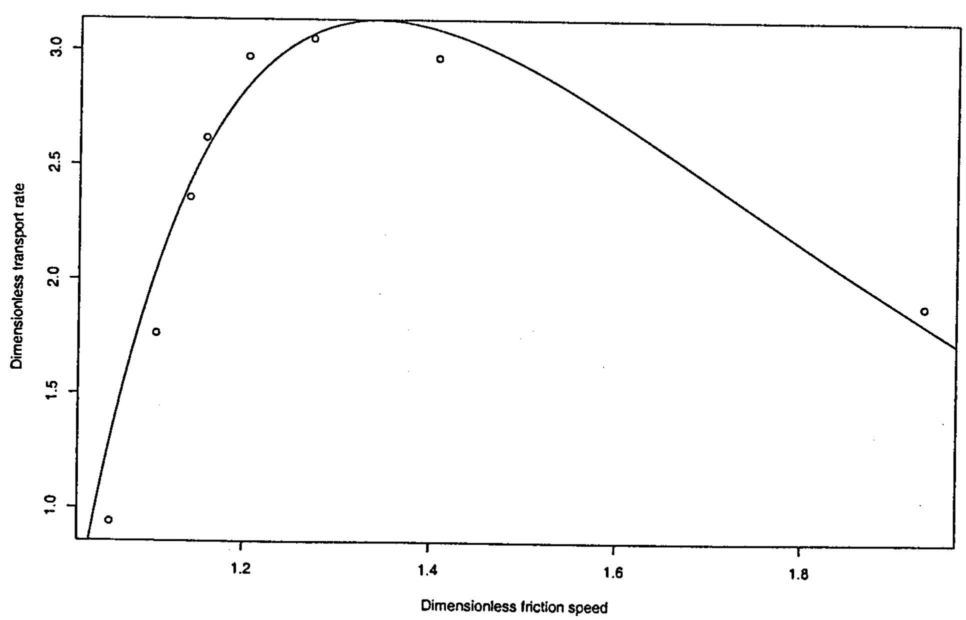

Figure 2: The dimensionless transport rate $Qg/(\rho U_*^3)$ plotted agains the dimensionless friction speed $V = U_*/U_{*c}$ for the experiment using natural sand with a typical grain size of 230 μm (indicated by $\circ$). The curve is the transport rate formula (7) with $\alpha = -1.78$, $\beta = 15.9$, and $\gamma = 0$.

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Conclusions

A simple explicit formula for the transport rate has been derived based on physical reasoning. The formula contains three parameters with a physical interpretation that can be estimated with measurements of transport rates. The formula fits data from wind tunnel experiments well.

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